



Building a Mathematical Model

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Using Excel, we will build a mathematical model based on data collected using easily obtained resources such as Styrofoam cups. We will then explore the mathematical model and its physical significance by introducing parameters for the model, examining measurement error via simulation, and interpreting the goodness-of-fit and linear regression curves. Assessment questions for students will also be covered. This workshop will demonstrate the power of Excel in visualizing data. No prior Excel experience is needed.

To view more interactive Excel spreadsheets or Excelets developed by teachers, see [Computation and Science for Teachers](#) (CAST), a joint program by the Pittsburgh Supercomputing Center and the Maryland Virtual High School.

Materials needed:

Excel file: http://academic.pgcc.edu/~ssinex/excelets/stacking_cups.xls

Styrofoam cups

30 cm metric rules (marked to the nearest millimeter)

Measuring the Stack Height of Nested Styrofoam Cups

Is there a relationship between the height of nested Styrofoam cups and the number of cups nested? If yes, elaborate on it.

Measure the stack heights of various stacks of nested Styrofoam cups. Record their heights to the nearest 0.1 cm. Record the cup volume: _____

Number of nested cups	Stack height (cm)

What is the dependent variable?

The collected data can be entered into the accompanying "Just Add Data" Excel spreadsheet on the *generate a model tab*

(http://academic.pgcc.edu/~ssinex/excelets/stacking_cups.xls). This will determine the line of best-fit by a linear regression performed on the data. This regression is your mathematical model! A measure of the goodness of fit for the regression is given by r^2 . A perfect fit of the model to the data would yield an $r^2 = 1$. Record your equation in terms of the variables studied (not x and y) and the value of r^2 here.

What does the slope represent in terms of the variables investigated?

What are the units of the slope?

What does the y-intercept represent in terms of the variables investigated?

What are the units of the y-intercept?

Now let's use the equation to make some predictions. What is the height of 150 nested Styrofoam cups? Explain the mathematics used to determine the height.

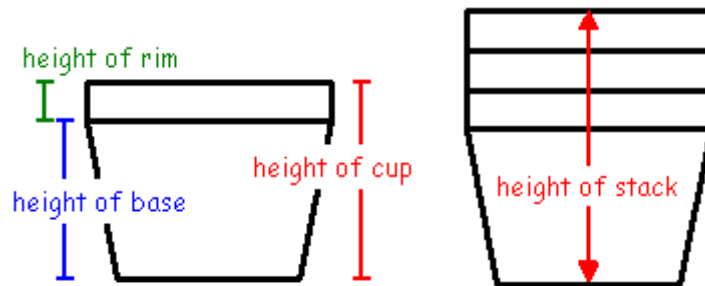
How many cups are required to obtain a height of 55 cm? Explain the mathematics used to determine this number.

You have a mathematical model of the nested cups of the form:

$$H = \text{slope} \times n + \text{intercept}$$

where H is the height of the nested stack and n is the number of cups.

Now go to the *simulate with model tab* and notice we have the capability to change a couple of parameters related to the cup - rim height and base height.



How does changing the height of the rim influence the regression line on the graph?

How does changing the height of the base influence the regression line on the graph?

Now rewrite your mathematical model to incorporate base height and rim height.

For the Styrofoam cups you used, what are the numerical values of the parameters based on the regression results:

height of the rim: _____ height of the base: _____

For the cups used, are these parameters above realistic? Can you tell anything about the uniformity of cup manufacturing? Explain.

Suppose we have to pack stacks of cups into cardboard boxes for shipping. How many cups could be in a stack to fit into a 61 cm long box? ...into a 100 cm long box?

To calculate the results above, what did you do algebraically?

Go to the *inverse function tab* and plot the data with the x and y variables reversed on the graph.

Write the equation of the inverse function in terms of the variables studied.

How are the original function and the inverse function related? (You may want to turn on the $y = x$ line on the graph to help.)

How does the goodness of fit behave for the inverse function? Explain.

Examining Error in the Model

Go to the *simulate errors* tab and you will find five different errors to investigate for this mathematical model. Error can come from measurement (the calibrated device and its use) and/or the manufacturing process of the cup.

Add each error, one at a time, to see how it influences the data and regression line. Set each back to no error before going to the next error.

Errors*	Slope of line	y-intercept	Scatter of data
Random measurement error			
Systematic measurement error			
Rim uniformity			
Base uniformity single stack			
Base uniformity multiple stacks			

*read the comment boxes on the Excellet for further explanation of the various errors

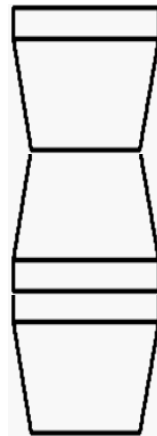
Do any of the errors above have a direction or bias (always positive or negative)? If so, explain.

When you made your measurements earlier, did the ruler and your use of it induce any error? Did you correct it? Could you correct it?

Assessment

Here is a model measured in centimeters: $H = 2.0n + 8.0$ What is the equation for the model in inches? (1 inch = 2.54 cm)

Using the model that you derived from the data, how would the model be modified to handle the stacking of the cups without nesting them (see illustration)? Explain and sketch a graph.



On a separate sheet of paper, complete the questions on the *assessment tab* and attached it to this handout.

Notes to the Instructors

This activity allows for the construction of a linear model using the stacking of nested Styrofoam cups of any size. Measurements should be made to the nearest 0.1 cm (1 mm) using centimeter rulers. Your model will be of the form:

$$H = \text{slope} \times n + \text{y-intercept}$$

Where H is the stack height in cm and n is the number of nested cups. The r^2 value for the model should be relatively high (close to 1.0) due to the consistent manufacturing of the Styrofoam cups, if students make the measurements carefully. If students are not careful, they usually add random scatter to the data and lower the r^2 value.

The slope of the graph represents the height of the rim of the cup, while the y-intercept is the height of the base of the cup. This leads to some student confusion as to why the graph has an intercept, since common sense says that zero cups have zero height. But this model is not a direct proportion like the stacking of cookies or blocks. The simulation tab on the Excellet will clear this up.

A common error with many 30 cm rulers is that the zero centimeter mark may not be at the end of the ruler. This induces a constant systematic error. Students may or may not notice this. Meter sticks do have the zero mark at the end.

Errors	Slope of line	y-intercept	Scatter of data
Random measurement error	(small changes)	(small changes)	r^2 decreases -adds scatter
Systematic measurement error	No effect	Positive - intercept increases Negative - intercept decreases	No effect
Rim uniformity	(small changes)	(small changes)	r^2 decreases -adds scatter
Base uniformity single stack	No effect	Taller - intercept increases Shorter - intercept decreases	No effect
Base uniformity multiple stacks	(small changes)	(small changes)	r^2 decreases -adds scatter

For further materials on mathematical modeling and developing interactive Excel spreadsheets, see the Developer's Guide to Excellets at <http://academic.pgcc.edu/~ssinex/exceleets>.